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Generalised Subtraction Procedure for Removing Power-Line Interference from ECG: Case of Powerline Frequency Deviation

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Abstract - **The present work continues the generalisation of the subtraction procedure, which removes the power-line interference without affecting the components intrinsic to the ECG. It is based on previous investigations, dealing separately with the cases of odd and even multiplicity/non-multiplicity between the sampling rate and the power-line frequency. The study proposes a common equations and algorithm for the cases of power-line frequency deviation. The theoretical conclusions are implemented in a program written in MATLAB environment. The algorithm and the program are tested by many particular cases and the introduced errors are evaluated. The work represents a suitable platform for accurate investigation, analysis and development of the subtraction procedure.**

Keywords – **Digital filtering, ECG filtering, Interference rejection.**

I. Introduction

The subtraction procedure for power-line (PL) interference removal from ECG signals [1, 10] has already shown high efficiency but continue to be subject of complementary investigations [2-10]. Its generalised structure consists of three main stages.

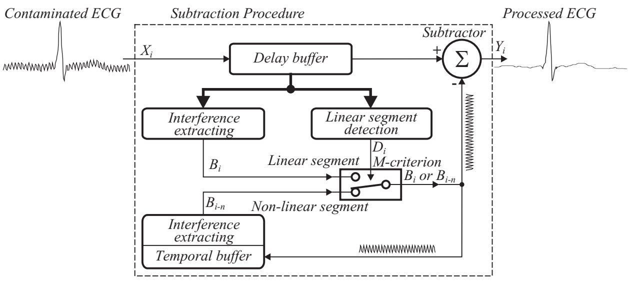

Fig. 1. Generalized structure of the subtraction procedure.

1. The first stage checks whether the ongoing sample X_i of the ECG signal belongs to a linear segment (usually contaminated by interference). The linearity is examined by the criterion Cr (called D-filter), which value must be less than a defined threshold M.

$$Cr < M \qquad (1)$$

2. When the ongoing sample belongs to linear segment, the procedure passes through the stage of interference extracting. The PL interference is removed with a non-recursive digital filter (called K-filter) of the type

$$Y_i = \sum_{j=-k}^{k} a_{i+j} X_{i+j} \qquad (2)$$

resulting in free of interference sample Y_i. Then, the ongoing sample of the interference B_i is simply calculated by subtracting the filtered Y_i sample from the non-filtered X_i

$$B_i = X_i - Y_i\,, \qquad (3)$$

and is stored in a temporal interference buffer.

3. If the ongoing sample X_i does not belong to a linear segment, the procedure passes through the stage of interference restoring, where ongoing sample of the interference B_i is calculated by the content of the temporal buffer. It is used to remove the interference component of the ongoing free of interference sample Y_i

$$Y_i = X_i - B_i\,, \qquad (4)$$

after that is stored back into the temporal buffer.

The temporal buffer keeps n preceding values of the PL interference B_{i-1}, B_{i-2}, …, B_{i-n}. The parameter $n = \Phi/F$ stands for the number of samples in one period of the PL interference (Φ is the sampling rate and F is the PL frequency). If Φ and F are multiple (case of multiplicity), n is integer. The multiplicity can be 'odd' $n = 2m+1$ or 'even' $n = 2m$.

In case of multiplicity the restored sample of the interference B_i just takes from the temporal buffer the value

$$B_i = B_{i-n}\,, \qquad (5)$$

which is phase locked with the ongoing interference sample.

When n is a real number (case of non-multiplicity), it is replaced in the equations with the rounded value n^* (the function used in the MATLAB environment is `round(Φ/F)`). In this case, Eq. (5) could not be applied, due to the phase difference between B_i and B_{i-n^*}. An additional filtering procedure (called B-filter) is introduced,

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which calculates the preceding interference sample by the content of the temporal buffer.

The first version of the subtraction procedure dealt with odd multiplicity, using non-recursive symmetric K-filter [1]. Later on, a similar K-filter was proposed for the case of even multiplicity [3, 4] and the theoretical base of the subtraction procedure was developed to overcome the ongoing PL frequency deviation, that is equivalent to a multiplicity divergence [5, 6, 7].

Previous investigations examined separately the cases of odd and even multiplicity/non-multiplicity between the sampling rate and the PL frequency. Recently, those both cases was united in [10] by generalised equation and common algorithm for a permanent frequency of the PL interference.

This work continues this generalisation, proposing a common algorithm for overcoming the ongoing PL frequency deviation.

II. Generalised Equations

The known symmetric non-recursive digital filters, used as a K-filter in the subtraction procedure is described by the equation

$$Y_i = \frac{1}{n}\left[\sum_{j=-m}^{m} X_{i+j} - \frac{c_m}{2}\left(X_{i-m} + X_{i+m}\right)\right]. \qquad (6)$$

The introduced in [10] coefficient $c_m = 2m+1-n$ expresses the mode of the multiplicity. It equals 0 for odd multiplicity and 1 for even multiplicity.

Their frequency responses is expressed by

$$K(f) = \frac{1}{n}\cdot\frac{\sin\frac{n\pi f}{\Phi}}{\sin\frac{\pi f}{\Phi}} S_C(f), \quad S_C(f) = \cos\frac{c_m \pi f}{\Phi}, \qquad (7)$$

where S_C takes the values 1 or $\cos(\pi f/\Phi)$ for odd and even multiplicities, respectively. The transfer coefficient $K(f)$ is unity at $f=0$. At $f=F$, $K(F) \equiv K_F$ is zero in case of multiplicity, while in the opposite case its value K_F differs from zero

$$K_F = \frac{1}{n^*}\cdot\frac{\sin\frac{n^*\pi F}{\Phi}}{\sin\frac{\pi F}{\Phi}}\cdot S_C, \quad S_C = \cos\frac{c_m \pi F}{\Phi}. \qquad (8)$$

For a case of non-multiplicity, [3] offers a modification of the filtered in linear segments ongoing sample Y^*_i by

$$Y_i^* = X_i - \frac{B_i}{1-K_F}, \quad B_i = X_i - Y_i \qquad (9)$$

and calculated the modify interference value B^*_i by

$$B_i^* = X_i - Y_i^* \qquad (10)$$

that is stored into the temporal interference buffer.

Further [6, 7], the interference sample B^*_i belonging to non-linear segment is restored using the generalised formula

$$B_i^* = B_{i-n^*}^* + \left(B_{i-(m-c_m)}^* - B_{i-(m+1)}^*\right)\frac{n^* K_F}{S_C^2\left(1+c_m\right)}. \qquad (11)$$

The linearity criterion Cr physically corresponds to the averaged within the PL period acceleration of the signal. Mathematically, the D-filter represents the curvature of the signal, which is expressed by the second derivative of the samples. First derivatives are taken by samples, which are spaced at one period of the PL frequency, thus eliminating the interference influence on the linearity evaluation.

For the case of non-multiplicity, [6] offers a modified D-filter

$$D^*_i = D_i + A_i\frac{D_F}{A_F}, \qquad (12)$$

where

$$D_i = \left(X_{i-\nu} + X_{i+\nu}\right)\left(1-k_n\right) + \left(X_{i-\nu-1} + X_{i+\nu+1}\right)k_n - 2X_i \qquad (13)$$

$$A_i = \frac{X_i}{2} - \left(X_{i+\mu} + X_{i-\mu}\right)\left(1-k_m\right)\frac{1}{4} - \left(X_{i+\mu+1} + X_{i-\mu-1}\right)k_m\frac{1}{4} \qquad (14)$$

$$\begin{aligned} D_F &= -4\sin^2\frac{\nu\pi F}{\Phi}\left(1-k_n\right) - 4\sin^2\frac{(\nu+1)\pi F}{\Phi}k_n; \\ A_F &= -\sin^2\frac{\mu\pi F}{\Phi}\left(1-k_m\right) - \sin^2\frac{(\mu+1)\pi F}{\Phi}k_m. \end{aligned} \qquad (15)$$

Here ν is the highest integer less than or equal to Φ/F, (in MATLAB environment the function `floor(Φ/F)` is used) and $k_n = \Phi/F - \nu$. Parameter μ is the highest integer less than or equal to $\Phi/(2F)$, (in MATLAB environment the function `floor(Φ/F/2)` is used), and $k_m = \Phi/(2F) - \mu$.

Since the linearity criterion does not depend on the type of multiplicity (odd or even), the Eq. (16) is common for both of them.

III. An Algorithm for Compensation the PL Frequency Deviation

The compensation of PL frequency variations requests D-, K- and B-filters to be currently remodified to face those variations. In [6] was shown that there is no need to remodify D-filter (its frequency response is close to zero within a range of normal PL frequency deviation around the rated value F). The remodifying of the K- and B-filters could be done just by recalculation the coefficient K_F (see Eqs. (9, 11)).

The algorithm offers coefficient K_F to be dynamically recalculated during stages of the interference removing from linear segments. According to Eq.(11), the coefficient K_{Fnew} is computed by:

$$K_{Fnew} = \frac{\left(B_i^* - B_{i-n^*}^*\right)S_C^2\left(1+c_m\right)}{n^*\left(B_{i-(m-c_m)}^* - B_{i-(m+1)}^*\right)}. \qquad (16)$$

```
LX=3200;              % Length of episode
                          %%% Constants calculating %%%
n = round(Q/F);       % Multiplicity
ni = floor(Q/F);      % Multiplicity
m = floor(n/2);       % Odd or even multiplicity
mu = floor(Q/F/2);    % Odd or Even multiplicity

Cm = 2*m+1-n;
Sc = cos(Cm*pi*F/Q);
KF0 = (sin(n*pi*F/Q)/sin(pi*F/Q))/n*Sc; % Initial value of KF
KF = KF0;
KFnew=KF;

kn = Q/F-ni;
km = Q/F/2-mu;
DF = -4*(sin((pi*F*ni)/Q))^2*(1-kn)-4*(sin((pi*F*(ni+1))/Q))^2*kn;
AF = -(sin(pi*F*mu/Q))^2*(1-km)-(sin(pi*F*(mu+1)/Q))^2*km;

                          %%% Algorithm %%%
for i=1+ni+1: 1: LX-ni-1;
    KFnew=KF;
    D(i) = (X(i-ni)+X(i+ni))*(1-kn)+(X(i-ni-1)+X(i+ni+1))*kn - 2*X(i);   %Linearity estimation
    A(i) = (2*X(i)-(X(i-mu)+X(i+mu))*(1-km)-(X(i-mu-1)+X(i+mu+1))*km)/4; % original K-filter
    Ds(i) = D(i)+A(i)*DF/AF;
    Cr = max(abs(Ds(i)),abs(Ds(i-1)));
    if Cr < M;                      % Linear segment
        Y(i)=-Cm/2*X(i-m)/n;        % Start of averaging
        for j=i-m: 1: i+m;
            Y(i)=Y(i)+X(j)/n;       % Averaging
        end
        Y(i)= Y(i)-Cm/2*X(i+m)/n;   % End of averaging
        B(i)=X(i)-Y(i);             % Interference estimation
        Ys(i)=X(i)-B(i)/(1-KF);     % Output sample modification for non-multiplied sampling
        Bs(i)=X(i)-Ys(i);           % Interference correction for non-multiplied sampling
       if abs(Bs(i-m+Cm)-Bs(i-m-1))>Bmin; %division zero protection
         KFnew = (Bs(i)-Bs(i-n))/((Bs(i-m+Cm)-Bs(i-m-1))*n)*Sc^2*(1+Cm);
       end
    else                            % Non-linear segment
        Bs(i)= Bs(i-n)+n*KF*(Bs(i-m+Cm)-Bs(i-m-1))/((1+Cm)*Sc^2); % Interference restoring
        Ys(i)=X(i)-Bs(i);           % Output sample estimation
    end
    if KFnew-KF>KFspd; %KF speed protection (rising)
        KFnew = KF+KFspd;
    end
    if KFnew-KF<-KFspd; %KF speed protection (falling)
        KFnew = KF-KFspd;
    end
    KF=KF*(2*n-1)/(2*n)+KFnew*1/(2*n);
    if KF > KFmax; %KF maximum protection
        KF = KFmax;
    end
    if KF < KFmin; %KF minimum protection
        KF = KFmin;
    end
    dFPL(i) = (KF0-KF)*sin(pi/n)*Q/pi;
end
```

Fig. 2. Program fragment of the subtraction procedure in Matlab environment. Equivalent notations used: **Q** ≡ Φ; **n** ≡ n^*; **ni** ≡ ν; **mu** ≡ μ; **Cm** ≡ c_m; **Sc** ≡ S_c; **KF** ≡ K_F; **KF0** ≡ K_{F0}; **KFmax** ≡ $K_{F\max}$; **KFmin** ≡ $K_{F\min}$; **KFspd** ≡ K_{Fspd}; **kn** ≡ k_n; **km** ≡ k_m; **DF** ≡ D_F; **AF** ≡ A_F; **Ds** ≡ D^*; **Bs** ≡ B^*; **Bmin** ≡ B_{min}; **Ys** ≡ Y^*; **dFPL** ≡ dF.

A program fragment of the procedure for MATLAB is shown in Fig. 2.

Some restrictions are introduced when the program has been written:

1. The K_{Fnew} calculation is protected against division by zero. The division is not performed if the denominator in Eq.(16) is less than $B_{\min}$ (normally the value of one bit);

2. The maximal speed K_{Fspd} of K_{Fnew} alteration is limited

$$\left|K_{Fnew} - K_F\right| \le K_{Fspd}, \quad K_{Fspd} = \frac{K_{F\max} - K_{F\min}}{2\Phi}, \quad (17)$$

where $K_{F\max}$ and $K_{F\min}$ are maximal and minimal values of the $K_{F\max}$ within the expected PL frequency deviation. That means that the full range of the expected deviation is allowed to be held within 2 *s*.

3. Since the PL frequency compensation is organized as a typical feedback control, the instability is avoided by a proportionally integral rule of adjustment. The new

calculated value KFnew by Eq. (16) is restored in Eqs. (9, 11) integrated within interval of 2 periods of PL interference.

$$K_F = \frac{2n-1}{2n} K_F + \frac{1}{2n} K_{Fnew} \,. \tag{18}$$

4. The restored coefficient K_F is restricted within the allowed range of PL frequency variation

$$K_{F\min} \le K_F \le K_{F\max} \tag{19}$$

IV. Evaluation of the PL Frequency Deviation

The calculated coefficient K_F could be used to evaluate the frequency of the PL interference. Considering F variable in Eq. (8), one may write for the first derivate of K_{BF}

$$\frac{dK_F}{dF} \approx S_C \frac{\pi}{\Phi} (1 - K_F) \sin^{-1} \frac{\pi F}{\Phi} \,. \tag{20}$$

Thus the PL frequency deviation may be expressed by the simple formula

$$dF \approx \frac{\Phi}{\pi S_C} (K_{F0} - K_F) \sin \frac{\pi F}{\Phi} , \tag{21}$$

where K_{F0} is the initial value of K_F.

V. Experimental Results

The subtraction procedure is tested with some AHA database signals for three different interference frequencies. The used complex criterion of linearity Cr is $|D^*_i| < M \wedge |D^*_{i-1}| < M$ [2]. The experiments are performed in the following sequence (see Fig. 3.a):

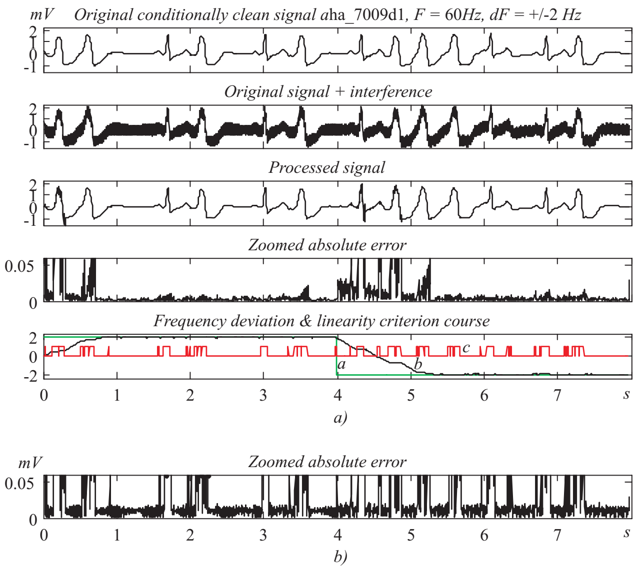

Fig. 3. Test with Φ = 250 *Hz*, *F* = 60 *Hz* and *dF* = ±2 *Hz*.

1. An episode, which is considered as a conditionally free of PL interference, is taken (*Original conditionally clean signal*). The duration is 8 s; the sampling rate is Φ = 250 Hz.

2. Synthesised PL interference with amplitude p = 1 mV is added to the conditionally clean signal that is shown in the second subplot (*Original signal + interference*). An abrupt change in PL frequency *F* from +*dF* to –*dF* is simulated in the middle of the epoch.

3. The Contaminated signal is subjected to the subtraction procedure and the filtered signal is shown in the third subplot (*Processed signal*).

4. The absolute difference between the filtered and the conditionally clean signal may be observed in the forth subplot (*Zoomed absolute error*).

5. The fifth subplot (*Frequency deviation & linearity criterion course*) shows the PL frequency deviation (curve *a* – green), the estimated according to equation (21) diversion of the PL frequency (curve *b* – black) and the criterion for linearity (curve *c* – red).

The result without PL frequency compensation is shown for comparison in Fig. 3b. The error is more than ten times higher, especially within the QRS complexes.

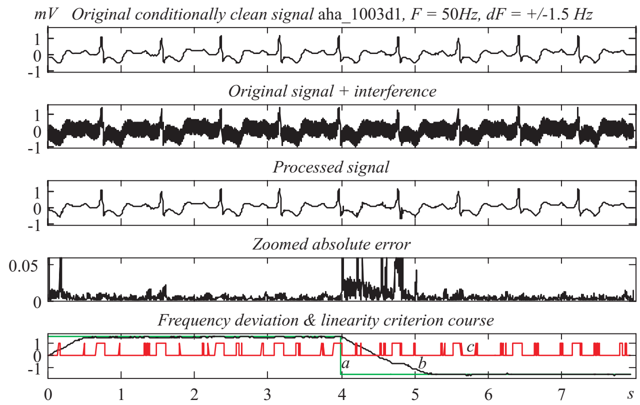

Fig. 4. Test with Φ = 250 *Hz*, *F* = 50 *Hz* and *dF* = ±1,5 *Hz*.

Fig. 4 and Fig. 5 show the same experiment, but with *F* = 50 *Hz*, *dF* = ±1,5 *Hz* and *F* = 16,7 *Hz*, *dF* = ±0,5 *Hz* respectively.

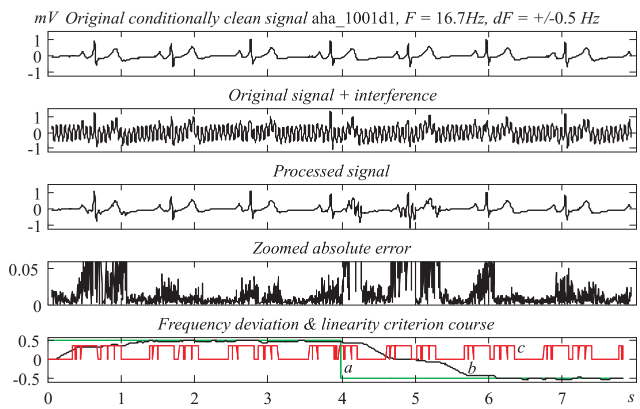

Fig. 5. Test with Φ = 250 *Hz*, *F* = 16,7 *Hz* and *dF* = ±0,5 *Hz*.

Fig. 6, Fig. 7 and Fig. 8 show the result of PL interference removing from old ECG records of 12 standard leads of our own database. The chosen episodes are contaminated by a real PL interference 50 *Hz*. The records are sampled with $\Phi = 400$ *Hz*. We define a range of PL interference frequency $dF = 1$ *Hz*, around the expected rated value of PL frequency $F = 50$ *Hz* for the record N0092(C2).

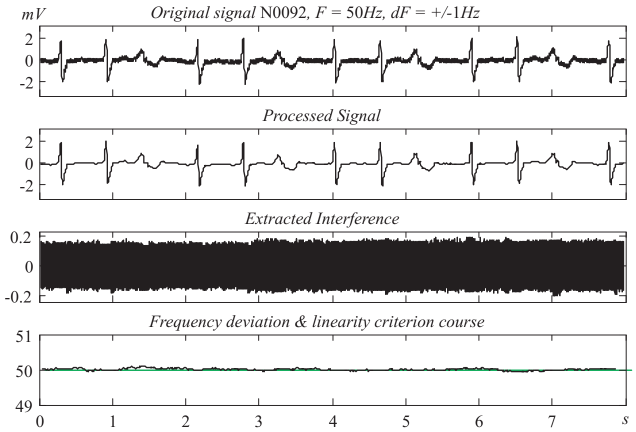

Fig. 6. Test with $\Phi = 400$ *Hz*, $F = 50$ *Hz* and $dF = \pm 1$ *Hz*.

For testing the stability of the PL frequency algorithm stability, we define an expected rated value of the PL frequency $F = 49{,}5$ *Hz* for record N0001(C1) and 50,5 *Hz* for record N0039(C1).

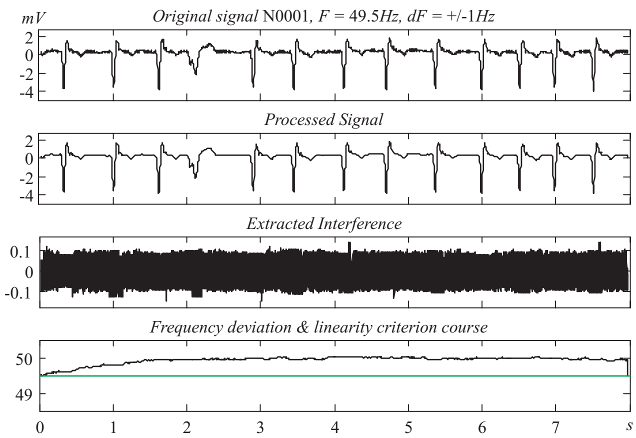

Fig. 7. Test with $\Phi = 400$ *Hz*, $F = 49{,}5$ *Hz* and $dF = \pm 1$ *Hz*.

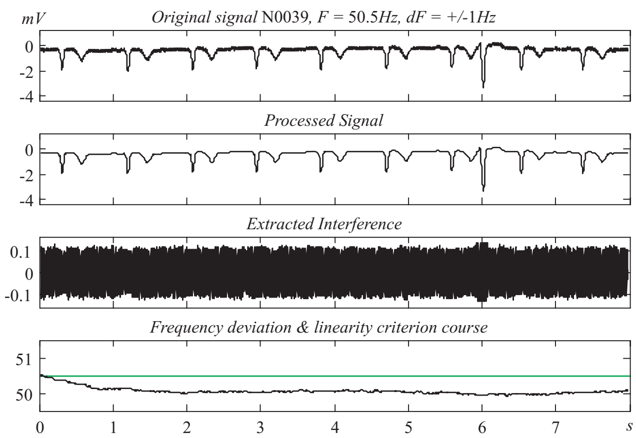

Fig. 8. Test with $\Phi = 400$ *Hz*, $F = 50{,}5$ *Hz* and $dF = \pm 1$ *Hz*.

VI. Discussions and Conclusions

This work is based on our previous investigations. It continues the generalisation of the subtraction procedure for the cases of PL frequency deviation, using uniformed equations thus providing a platform for further studies and improvements.

The algorithm and the program written in MATLAB environment are tested with many particular cases. The results obtained lead to the following conclusions:

Linear segment detection in signal contaminated by interference with frequency deviations is a priori very difficult process. However, the detection is lightened due to the close to zero frequency response of the D-filter within a range of normal PL frequency deviation around its rated value – no need to remodify it.

The time necessary to be reached the stationary value of the changed PL frequency is about 1 *s* (abstracting by the time periods of non-linear segments). The error committed is less than 25 μ*V*. The error becomes higher and reaches a maximal value of about 50 μ*V* with PL frequency of 16,7±0,5 *Hz*, where the linear segment detection is embarrassed.

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